LEVI-CIVITA CONNECTIONS OF FLAG MANIFOLDS

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Abstract. For any flag manifold G/T we obtain an explicit expression of its Levi-Civita connection with respect to any invariant Riemannian metric.

1. Introduction

Let G/T be a flag manifold, where T is a maximal torus of a compact semi-simple Lie group G. In this case we obtain an explicit formula of its Levi-Civita connection (in terms of the root decomposition for the Lie algebra $\mathfrak{g}$ of G) with respect to any invariant Riemannian metric. It is possible to realize this formula, for example, in the case of any classical simple Lie group G. In this paper it is done for $SU(n)$.

This result may prove useful in solving different problems. For instance, it enables us to determine whether a given metric f-structure (f, g) on G/T belongs to the main classes of generalized Hermitian geometry (see, for example, [2] and [1]).

2. Levi-Civita connections of flag manifolds

In this paper we consider a flag manifold G/T, where T is a maximal torus of a compact semi-simple Lie group G. Let $\mathfrak{g}$ and $\mathfrak{t}$ be the corresponding Lie algebras of G and T. G/T is a reductive homogeneous space, its reductive decomposition being $\mathfrak{g} = \mathfrak{t} \oplus \mathfrak{m}$, where $\mathfrak{m}$ is an orthogonal complement of $\mathfrak{t}$ in $\mathfrak{g}$ with respect to the Killing form B of $\mathfrak{g}$. Denote by $\mathfrak{g}^{\mathbb{C}}$ and $\mathfrak{t}^{\mathbb{C}}$ the complexifications of $\mathfrak{g}$ and $\mathfrak{t}$. Then $\mathfrak{t}^{\mathbb{C}}$ is a Cartan subalgebra of $\mathfrak{g}^{\mathbb{C}}$ and we denote by R the root system of $\mathfrak{g}^{\mathbb{C}}$ with respect to $\mathfrak{t}^{\mathbb{C}}$. In this way we have the root decomposition

$$\mathfrak{g}^{\mathbb{C}} = \mathfrak{t}^{\mathbb{C}} \oplus \sum_{\alpha \in R} \mathfrak{g}^{\alpha}. \tag{1}$$

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Let $\Pi = \{\alpha_1, \alpha_2, \dots, \alpha_l\}$ be a basis of R. Denote by R^+ the set of all positive roots and by R^- the set of all negative roots. In this paper the following notation will be used:

$$|\alpha| = \begin{cases} \alpha, & \text{if } \alpha \in R^+, \\ -\alpha, & \text{if } \alpha \in R^-. \end{cases}$$

Recall that we can consider the lexicographic order on R: $\gamma = \sum_{i=1}^n \gamma_i \alpha_i$ is said to be greater than $\delta = \sum_{i=1}^n \delta_i \alpha_i$ ($\gamma > \delta$) if the first nonzero coefficient $\gamma_k - \delta_k$ in the decomposition $\gamma - \delta = \sum_{i=1}^n (\gamma_i - \delta_i)\alpha_i$ is positive. If $\gamma - \delta \in R$ then $\gamma > \delta$ if and only if $\gamma - \delta \in R^+$.

It is well-known that in the case under consideration the reductive complement $\mathfrak{m}$ can be decomposed into the direct sum of 2-dimensional $Ad(T)$-modules $\mathfrak{m}^\alpha$ which are mutually non-equivalent:

$$\mathfrak{m} = \sum_{\alpha \in R^+} \mathfrak{m}^\alpha, \text{ where } \mathfrak{m}^\alpha = \mathfrak{g}^\alpha \oplus \mathfrak{g}^{-\alpha}.$$

Therefore, any invariant Riemannian metric $g = \langle \cdot, \cdot \rangle$ on G/T is given by

$$g = \langle \cdot, \cdot \rangle = \sum_{\alpha \in R^+} c_\alpha (\cdot, \cdot) \mid_{\mathfrak{g}^\alpha \oplus \mathfrak{g}^{-\alpha}}, \tag{2}$$

where $c_\alpha > 0$, $(\cdot, \cdot)$ is the negative of the Killing form B of the Lie algebra $\mathfrak{g}$.

In this paper we will need the following result.

Theorem 1. [3] *Let (M, g) be a Riemannian manifold, $M = G/H$ a reductive homogeneous space with the reductive decomposition $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{m}$. Then the Levi-Civita connection with respect to g can be expressed in the form*

$$\nabla_X Y = \frac{1}{2}[X, Y]_{\mathfrak{m}} + U(X, Y), \tag{3}$$

where U is a symmetric bilinear mapping $\mathfrak{m} \times \mathfrak{m} \to \mathfrak{m}$ defined by the formula

$$2g(U(X, Y), Z) = g(X, [Z, Y]_{\mathfrak{m}}) + g([Z, X]_{\mathfrak{m}}, Y), \; X, Y, Z \in \mathfrak{m}. \tag{4}$$

We can consider (4) as an equation of variable U. Let us try to solve this equation in the case of an arbtrary flag manifold G/T.

We begin with obtaining an important preliminary result. Consider $X_\gamma \in \mathfrak{g}^\gamma$, $Y_\delta \in \mathfrak{g}^\delta$, γ, $\delta \in R$. In the view of (2), (4) takes the following form:

$$\begin{aligned} 2\sum_{\alpha\in R^+} c_\alpha(U(X_\gamma,Y_\delta)_{\mathfrak{g}^\alpha\oplus\mathfrak{g}^{-\alpha}}, Z_{\mathfrak{g}^\alpha\oplus\mathfrak{g}^{-\alpha}}) \\ = \sum_{\alpha\in R^+} c_\alpha((X_\gamma)_{\mathfrak{g}^\alpha\oplus\mathfrak{g}^{-\alpha}}, [Z,Y_\delta]_{\mathfrak{g}^\alpha\oplus\mathfrak{g}^{-\alpha}}) \\ + \sum_{\alpha\in R^+} c_\alpha([Z,X_\gamma]_{\mathfrak{g}^\alpha\oplus\mathfrak{g}^{-\alpha}}, (Y_\delta)_{\mathfrak{g}^\alpha\oplus\mathfrak{g}^{-\alpha}}). \quad (5)\end{aligned}$$

Obviously, the right-hand side of this equation is equal to

$$c_{|\gamma|}(X_\gamma, [Z,Y_\delta]_{\mathfrak{g}^{|\gamma|}\oplus\mathfrak{g}^{-|\gamma|}}) + c_{|\delta|}([Z,X_\gamma]_{\mathfrak{g}^{|\delta|}\oplus\mathfrak{g}^{-|\delta|}}, Y_\delta).$$

Let $Z=\sum_{\alpha\in R} Z_\alpha$, where $Z_\alpha = Z_{\mathfrak{g}^\alpha}$. Note that

$$[Z,Y_\delta]_{\mathfrak{m}} = [\sum_{\alpha\in R} Z_\alpha, Y_\delta]_{\mathfrak{m}} = \sum_{\alpha\in R}[Z_\alpha,Y_\delta]_{\mathfrak{m}} = \sum_{\alpha,\alpha+\delta\in R}[Z_\alpha,Y_\delta],$$

and, evidently, $[Z_\alpha,Y_\delta]=[Z,Y_\delta]_{\mathfrak{g}^{\alpha\oplus\delta}}$. It is easy to see that

$$\begin{aligned}(X_\gamma,[Z,Y_\delta]_{\mathfrak{g}^{|\gamma|}\oplus\mathfrak{g}^{-|\gamma|}}) &= \left(X_\gamma, \left(\sum_{\alpha,\alpha+\delta\in R}[Z_\alpha,Y_\delta]\right)_{\mathfrak{g}^{|\gamma|}\oplus\mathfrak{g}^{-|\gamma|}}\right) \\ &= \left(X_\gamma, \left(\sum_{\alpha,\alpha+\delta\in R}[Z_\alpha,Y_\delta]\right)_{\mathfrak{g}^{-\gamma}}\right).\end{aligned}$$

If $\left(\sum_{\alpha,\alpha+\delta\in R}[Z_\alpha,Y_\delta]\right)_{\mathfrak{g}^{-\gamma}} \neq 0$, then there exists such $\alpha\in R$ that $\alpha+\delta=-\gamma$. In other words, $\alpha=-\gamma-\delta\in R$. Therefore,

$$c_{|\gamma|}(X_\gamma,[Z,Y_\delta]_{\mathfrak{g}^{|\gamma|}\oplus\mathfrak{g}^{-|\gamma|}}) = \begin{cases} 0, & \text{if } \gamma+\delta\notin R, \\ c_{|\gamma|}(X_\gamma,[Z_{-\gamma-\delta},Y_\delta]), & \text{if } \gamma+\delta\in R. \end{cases}$$

Arguing as above, one can prove that

$$c_{|\delta|}([Z,X_\gamma]_{\mathfrak{g}^{|\delta|}\oplus\mathfrak{g}^{-|\delta|}}, Y_\delta) = \begin{cases} 0, & \text{if } \gamma+\delta\notin R, \\ c_{|\delta|}([Z_{-\delta-\gamma},X_\gamma],Y_\delta), & \text{if } \gamma+\delta\in R. \end{cases}$$

Hence, if $\gamma+\delta\notin R$, (4) is transformed into

$$2g(U(X_\gamma,Y_\delta),Z)=0$$

for any $Z\in\mathfrak{m}$. Thus, if $\gamma+\delta\notin R$, then $U(X_\gamma,Y_\delta)=0$.

If $\gamma+\delta\in R$, then (5) is equivalent to

$$\begin{aligned}2\sum_{\alpha\in R^+} c_\alpha(U(X_\gamma,Y_\delta)_{\mathfrak{g}^\alpha\oplus\mathfrak{g}^{-\alpha}}, Z_{\mathfrak{g}^\alpha\oplus\mathfrak{g}^{-\alpha}}) \\ = c_{|\gamma|}(X_\gamma,[Z_{-\gamma-\delta},Y_\delta]) + c_{|\delta|}([Z_{-\delta-\gamma},X_\gamma],Y_\delta).\end{aligned}$$

By the properties of the Killing form we obtain

$$\sum_{\substack{\alpha\in R^+\\ \alpha\neq|\gamma+\delta|}} (2c_\alpha U(X_\gamma,Y_\delta)_{\mathfrak{g}^\alpha\oplus\mathfrak{g}^{-\alpha}}, Z_{\mathfrak{g}^\alpha\oplus\mathfrak{g}^{-\alpha}}) + (2c_{|\gamma+\delta|}U(X_\gamma,Y_\delta)_{\mathfrak{g}^{|\gamma+\delta|}\oplus\mathfrak{g}^{-|\gamma+\delta|}} - c_{|\gamma|}[Y_\delta,X_\gamma] - c_{|\delta|}[X_\gamma,Y_\delta], Z_{\mathfrak{g}^{|\gamma+\delta|}\oplus\mathfrak{g}^{-|\gamma+\delta|}}) = 0.$$

Since $\mathfrak{m}^\alpha$ is orthogonal to $\mathfrak{m}^\beta$ with respect to the Killing form of $\mathfrak{g}$ (α, $\beta\in R^+$, $\alpha\neq\beta$), we have

$$(2\sum_{\substack{\alpha\in R^+\\ \alpha\neq|\gamma+\delta|}} c_\alpha U(X_\gamma,Y_\delta)_{\mathfrak{m}^\alpha} + 2c_{|\gamma+\delta|}U(X_\gamma,Y_\delta)_{\mathfrak{m}^{|\gamma+\delta|}} - (c_{|\gamma|}-c_{|\delta|})[Y_\delta,X_\gamma], Z) = 0$$

for any $Z\in\mathfrak{m}$. This yields that

$$2\sum_{\substack{\alpha\in R^+\\ \alpha\neq|\gamma+\delta|}} c_\alpha U(X_\gamma,Y_\delta)_{\mathfrak{m}^\alpha} + 2c_{|\gamma+\delta|}U(X_\gamma,Y_\delta)_{\mathfrak{m}^{|\gamma+\delta|}} - (c_{|\gamma|}-c_{|\delta|})[Y_\delta,X_\gamma]$$

(and, consequently, any of its projections onto $\mathfrak{m}^\alpha$, $\alpha\in R^+$) is equal to 0. We have proved the following result.

Lemma 1. *Let G/T be a flag manifold with the root decomposition (1). Then for any $X_\gamma\in\mathfrak{g}^\gamma$, $Y_\delta\in\mathfrak{g}^\delta$, where γ, $\delta\in R$, we have*

$$U(X_\gamma,Y_\delta) = \begin{cases} \frac{c_{|\gamma|}-c_{|\delta|}}{2c_{|\gamma+\delta|}}[Y_\delta,X_\gamma], & \textit{if } \gamma+\delta\in R,\\ 0, & \textit{if } \gamma+\delta\notin R. \end{cases} \tag{6}$$

This lemma enables us to obtain the similar expression for $U(X,Y)$ in the case of any $X=\sum_{\alpha\in R}X_\alpha$ and $Y=\sum_{\beta\in R}Y_\beta$ in $\mathfrak{m}$. As U is bilinear, application of (6) gives us

$$U(X,Y) = \sum_{\alpha,\beta\in R} U(X_\alpha,Y_\beta) = \sum_{\alpha,\beta,\alpha+\beta\in R} \frac{c_{|\alpha|}-c_{|\beta|}}{2c_{|\alpha+\beta|}}[Y_\beta,X_\alpha]. \tag{7}$$

For any $\alpha,\beta\in R$ such that $\alpha+\beta\in R$ we group together terms with the coefficient $\frac{c_{|\alpha|}-c_{|\beta|}}{2c_{|\alpha+\beta|}}$. In this way we obtain the sum of the following summands

$$\frac{c_{|\alpha|}-c_{|\beta|}}{2c_{|\alpha+\beta|}}Z_\alpha^\beta,$$

where

$$Z_\alpha^\beta = [Y_\beta,X_\alpha] + [X_\beta,Y_\alpha] + [Y_{-\beta},X_{-\alpha}] + [X_{-\beta},Y_{-\alpha}],\ \alpha,\beta\in R. \tag{8}$$

However, $Z_\alpha^\beta = Z_{-\alpha}^{-\beta} = Z_\beta^\alpha = Z_{-\beta}^{-\alpha}$, which implies that there is a need to restrict the range of α and β. Certainly, (7) is equivalent to

$$U(X,Y) = \frac{1}{4}\sum_{\alpha,\beta\in R} \frac{c_{|\alpha|} - c_{|\beta|}}{2c_{|\alpha+\beta|}} Z_\alpha^\beta,$$

but this formula is definitely not the most convenient since there are repetitions of summands. Luckily, it is easy to establish a condition which makes it possible to select one pair of roots out of four pairs (α,β), (β,α), $(-\alpha,-\beta)$, $(-\beta,-\alpha)$.

Lemma 2. *For any $\alpha,\beta \in R$ there exists only one pair $(a_1,a_2) \in \{(\alpha,\beta),(\beta,\alpha),(-\alpha,-\beta),(-\beta,-\alpha)\}$ such that $|a_1| < a_2$.*

Proof. The condition $|a_1| < a_2$ presupposes that $a_2 \in R^+$. Obviously, $|a_1| < a_2$ if and only if $-a_2 < a_1 < a_2$.

Such a pair can be chosen as follows.

Set $a_1 = \alpha$, $a_2 = \beta$. If $a_2 \in R^-$, set a_1 equal to $-a_1$ and a_2 equal to $-a_2$. Thus we have $a_2 \in R^+$. Now let us check if $a_1 < a_2$. If this condition is not satisfied, set a_2 equal to a_1 and a_1 equal to a_2. It remains to verify if $a_1 > -a_2$. If this is true, the desired pair (a_1,a_2) is obtained , otherwise we choose $(-a_2,-a_1)$.

The uniqueness of this pair can be proved as follows. Without loss of generality, suppose that $|\alpha| < \beta$, that is, $-\beta < \alpha < \beta$. Then (β,α) satisfies $\beta > \alpha$ and for $(-\alpha,-\beta)$ we have $-\alpha > -\beta$ which means that these two pairs do not satisfy the stipulated condition. The pair $(-\beta,-\alpha)$ should satisfy $\alpha < -\beta < -\alpha$ and this contradicts the assumption made above. □

In the view of this lemma we have

$$U(X,Y) = \sum_{\substack{\alpha,\beta,\alpha+\beta\in R,\\ |\alpha|<\beta\in R^+}} \frac{c_{|\alpha|} - c_{|\beta|}}{2c_{|\alpha+\beta|}} Z_\alpha^\beta \tag{9}$$

(Z_α^β is determined by means of (8)).

Consider different cases for $\alpha,\beta \in R$. β always belongs to R^+ and α can be selected from both R^+ and R^-.

If $\alpha \in R^+$, $\beta \in R^+$ then the conditions $\alpha+\beta \in R$ and $|\alpha| < \beta$ can be replaced by the conditions $\alpha+\beta \in R^+$ and $\alpha < \beta$ respectively.

If $\alpha \in R^-$, $\beta \in R^+$ then $|\alpha| < \beta$ is equivalent to $-\alpha < \beta$. If $\alpha+\beta \in R$ then $-\alpha < \beta$ can be substituted for the condition $\alpha+\beta \in R^+$.

Therefore, the right-hand side of (9) is transformed into

$$\sum_{\substack{\alpha,\beta,\alpha+\beta\in R^+,\\ \alpha<\beta}} \frac{c_\alpha - c_\beta}{2c_{\alpha+\beta}} Z_\alpha^\beta + \sum_{-\alpha,\beta,\alpha+\beta\in R^+} \frac{c_{-\alpha} - c_\beta}{2c_{\alpha+\beta}} Z_{-\alpha}^\beta$$

$$= \sum_{\substack{\alpha,\beta,\alpha+\beta\in R^+,\\ \alpha<\beta}} \frac{c_\alpha - c_\beta}{2c_{\alpha+\beta}} Z_\alpha^\beta + \sum_{\alpha,\beta,\beta-\alpha\in R^+} \frac{c_\alpha - c_\beta}{2c_{\beta-\alpha}} Z_\alpha^\beta.$$

Thus, the following theorem is proved.

Theorem 2. *Let G/T be a flag manifold with the root decomposition (1). Then for any X, $Y \in \mathfrak{m}$ we have*

$$U(X,Y) = \sum_{\substack{\alpha,\beta,\alpha+\beta\in R^+,\\ \alpha<\beta}} \frac{c_\alpha - c_\beta}{2c_{\alpha+\beta}} Z_\alpha^\beta + \sum_{\alpha,\beta,\beta-\alpha\in R^+} \frac{c_\alpha - c_\beta}{2c_{\beta-\alpha}} Z_\alpha^\beta, \tag{10}$$

where $Z_\alpha^\beta = [Y_\beta, X_\alpha] + [X_\beta, Y_\alpha] + [Y_{-\beta}, X_{-\alpha}] + [X_{-\beta}, Y_{-\alpha}],\ \alpha, \beta \in R$.

3. Examples

As an example, let us consider the flag manifold $G/T = SU(n+1)/T$ ($n \geq 2$), where T is a maximal torus of $SU(n+1)$.

In this case $\mathfrak{g} = \mathfrak{sl}(n+1, \mathbb{C})$. The root system of $SU(n+1)$ with respect to $\mathfrak{t}$ is

$$R = A_n = \{\varepsilon_i - \varepsilon_j \mid i \neq j,\ 1 \leq i, j \leq n+1\},$$

its basis being

$$\{\alpha_i = \varepsilon_i - \varepsilon_{i+1}\}_{1\leq i\leq n}.$$

The set of all positive roots in this case is

$$R^+ = \{\varepsilon_i - \varepsilon_j \mid 1 \leq i < j \leq n\}.$$

An arbitrary positive root $\alpha = \varepsilon_i - \varepsilon_j$, where $i < j$, is decomposed into the sum of basis vectors as follows:

$$\alpha = \varepsilon_i - \varepsilon_j = \alpha_i + \alpha_{i+1} + \cdots + \alpha_j.$$

It is easy to see that $\alpha = \varepsilon_i - \varepsilon_j < \beta = \varepsilon_k - \varepsilon_l$ ($\alpha, \beta \in R^+$) if and only if $i > k$.

Take $\alpha = \varepsilon_i - \varepsilon_j$, $\beta = \varepsilon_k - \varepsilon_l \in R^+$, where $i < j$, $k < l$.

$\alpha+\beta \in R^+$ if and only if either $i < j = k < l$ (hence $\alpha+\beta = \varepsilon_i - \varepsilon_l$), or $k < i = l < j$ (hence $\alpha + \beta = \varepsilon_k - \varepsilon_j$). Note that in the first case $\alpha > \beta$ and in the second case $\beta > \alpha$.

$\beta-\alpha \in R^+$ if and only if either $i = k < j < l$ (hence $\beta-\alpha = \varepsilon_j - \varepsilon_l$), or $k < i < j = l$ (hence $\beta - \alpha = \varepsilon_k - \varepsilon_i$).

It is not difficult to show that $Z^{\beta}_{\alpha} = [X_{\mathfrak{m}^{\beta}}, Y_{\mathfrak{m}^{\alpha}}] + [Y_{\mathfrak{m}^{\beta}}, X_{\mathfrak{m}^{\alpha}}]$ for any $\alpha, \beta \in R^+$.

Therefore, in the case of $SU(n+1)/T_{max}$ ($n \geq 2$) (10) takes form

$$
\begin{aligned}
&U(X,Y)\\
&= \sum_{1\le i<j<k\le n+1} \frac{c_{\varepsilon_j-\varepsilon_k} - c_{\varepsilon_i-\varepsilon_j}}{2c_{\varepsilon_i-\varepsilon_k}}([X_{\mathfrak{m}^{\varepsilon_i-\varepsilon_j}}, Y_{\mathfrak{m}^{\varepsilon_j-\varepsilon_k}}] + [Y_{\mathfrak{m}^{\varepsilon_i-\varepsilon_j}}, X_{\mathfrak{m}^{\varepsilon_j-\varepsilon_k}}])\\
&+ \sum_{1\le i<j<k\le n+1} \frac{c_{\varepsilon_i-\varepsilon_j} - c_{\varepsilon_i-\varepsilon_k}}{2c_{\varepsilon_j-\varepsilon_k}}([X_{\mathfrak{m}^{\varepsilon_i-\varepsilon_k}}, Y_{\mathfrak{m}^{\varepsilon_i-\varepsilon_j}}] + [Y_{\mathfrak{m}^{\varepsilon_i-\varepsilon_k}}, X_{\mathfrak{m}^{\varepsilon_i-\varepsilon_j}}])\\
&+ \sum_{1\le i<j<k\le n+1} \frac{c_{\varepsilon_j-\varepsilon_k} - c_{\varepsilon_i-\varepsilon_k}}{2c_{\varepsilon_i-\varepsilon_j}}([X_{\mathfrak{m}^{\varepsilon_i-\varepsilon_k}}, Y_{\mathfrak{m}^{\varepsilon_j-\varepsilon_k}}] + [Y_{\mathfrak{m}^{\varepsilon_i-\varepsilon_k}}, X_{\mathfrak{m}^{\varepsilon_j-\varepsilon_k}}]).
\end{aligned}
\tag{11}
$$

As a particular case, let us consider the flag manifold $SU(3)/T_{max}$. The set of all positive roots is

$$R^+ = \{\alpha_1 = \varepsilon_1 - \varepsilon_2,\ \alpha_2 = \varepsilon_1 - \varepsilon_3,\ \alpha_3 = \varepsilon_2 - \varepsilon_3\}.$$

In order to obtain a more compact formula denote c_{α_i} by c_i and $\mathfrak{m}^{\alpha_i}$ by $\mathfrak{m}_i$. We also agree to write X_i instead of $X_{\mathfrak{m}_i}$.

Therefore, in the case of $SU(3)/T_{max}$, using the notations introduced above, we can rewrite (11) as follows:

$$
\begin{aligned}
U(X,Y) &= \frac{c_3 - c_2}{2c_1}([X_2, Y_3] + [Y_2, X_3])\\
&\quad + \frac{c_3 - c_1}{2c_2}([X_1, Y_3] + [Y_1, X_3]) + \frac{c_2 - c_1}{2c_3}([X_1, Y_2] + [Y_1, X_2]).
\end{aligned}
$$

Actually, this result is well-known (see, for example, [4]).

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